Chopping and Nodding for Mid-Infrared Astronomy

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Observations in the mid-infrared (or equivalently the *thermal infrared*, so called because at these wavelengths the thermal radiation from the atmosphere and the telescope are significant) are often done in chop/nod mode wherein one points the telescope at a position slightly off the target and moves the telescope field of rapidly between the source position and a sky position, usually by rocking the secondary mirror at a set frequency, and then offsets the telescope to the other side of the position to repeat the process now sampling the sky on the other side of the central position. This is illustrated in Figures 1 and 2 below. However please note that the angles over which chopping takes place are small: at Gemini the maximum chop amplitude is 15 arc-seconds, which is about the same as the separation of the Mizar A/B binary star pair in Ursa Major–a large enough separation to be seen by those with good eyesight, but still small on the sky. In the Figures the angles are drawn as far larger than the real angles on the sky for ease of labeling the different positions.

This document is intended to show how this method of observing works to remove the sky and telescope backgrounds to give just the emission from the target.

As an aside, in cases where the background contribution is changing only slowly chopping is not needed and observation can be carried out in a simple nod mode. That situation is closely analogous to what is discussed here, but is less complicated because only two observation positions on the sky and one view of the telescope need to be considered. The pure nod case will not be considered here. Pure nodding is often used in L-band ($\approx$3.5 μm) and in M-band ($\approx$5 μm) to remove the thermal background, and it is used at shorter wavelengths including in optical spectroscopy to remove airglow or other non-thermal sky background radiation.

To understand what is happening in chop/nod observations one has to keep straight three separate sources of emission:

(1) a source, seen in chop position A/nod position A and chop position B/nod position B (it may be still in the field of view of the detector in the other nod position, but that is not of concern here);

(2) the telescope radiation, seen in both the chop A and chop B positions–chop A will be assumed to be looking at telescope from the “right” position, and chop B will be assumed to be looking at the telescope from the “left” position, which two views are not exactly the same but which are fixed as the telescope is moved to point at different positions on the sky;

(3) the sky radiation–actually from the Earth’s atmosphere–which is viewed at three different positions which I will denote “A”, “B”, and “C” here.

The source contribution is assumed to be constant here. In many cases when observing in the thermal infrared the emission from the telescope and from the atmosphere is much larger than the emission from the source. As the secondary mirror is moving only slightly the views of the telescope from the “left” and “right” positions will be similar, but for example any edges of objects or other positions where there is contrast in the field of view will shift somewhat between the two chop positions.

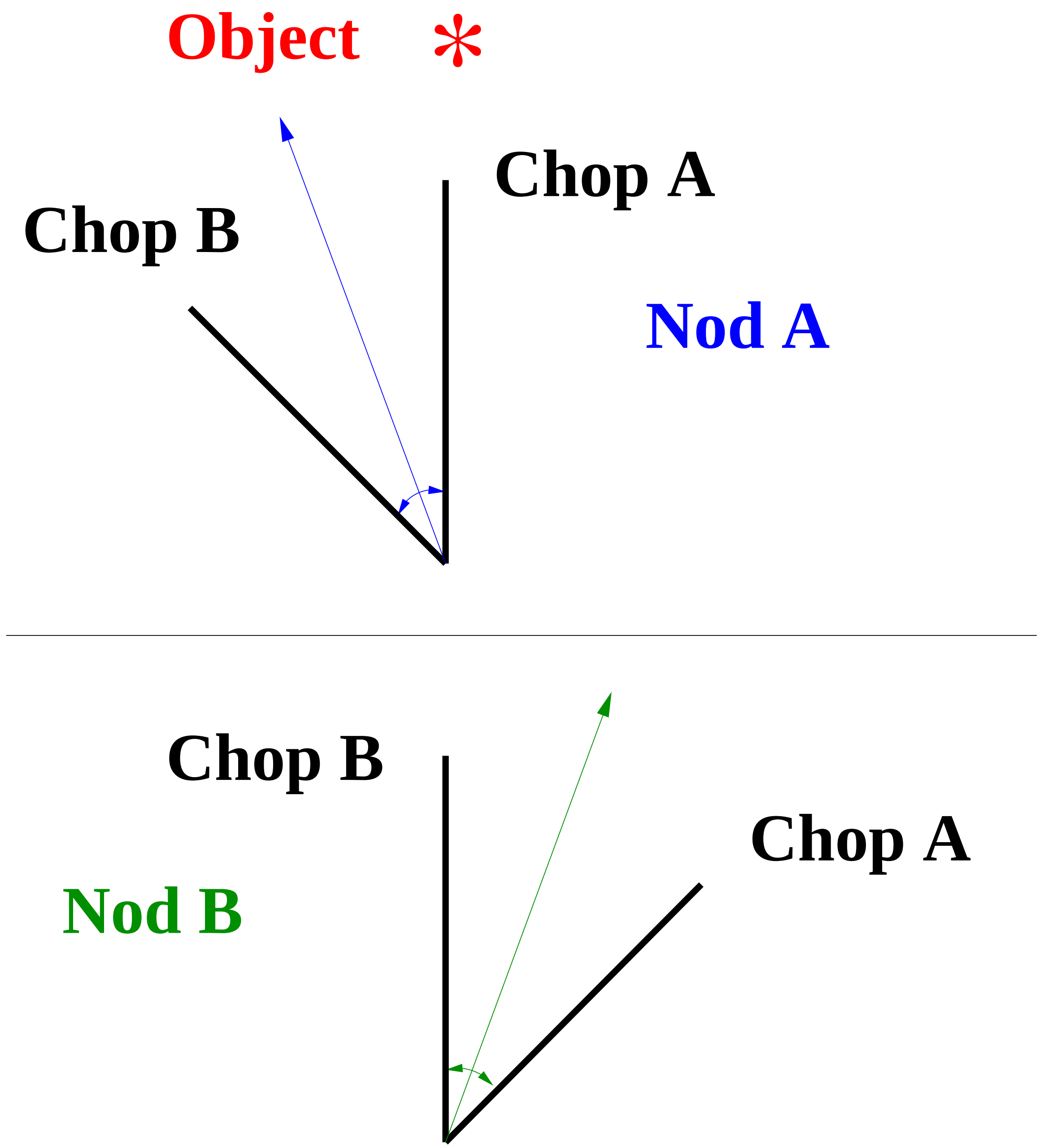

Figure 1 – Illustration of chop/nod observations. The upper part of the Figure shows the telescope pointing in nod A, off to one side of the object (blue arrow) and the black lines indicate the two pointings of the telescope while chopping at this position. The lower part shows the telescope pointing in nod B (green arrow) now on the other side of the object, and the associated two chop positions. As long as the nod motion is exactly the same direction and amplitude as the chop motion then the source position on the detector in chop A/nod A will be the same as the source position on the detector in chop B/nod B.

Note that the angles involved are considerably exaggerated; the actual chop angle is so small that if the pairs of black lines were drawn with this angle between them one would not be able to see the lines as separated on the diagram.

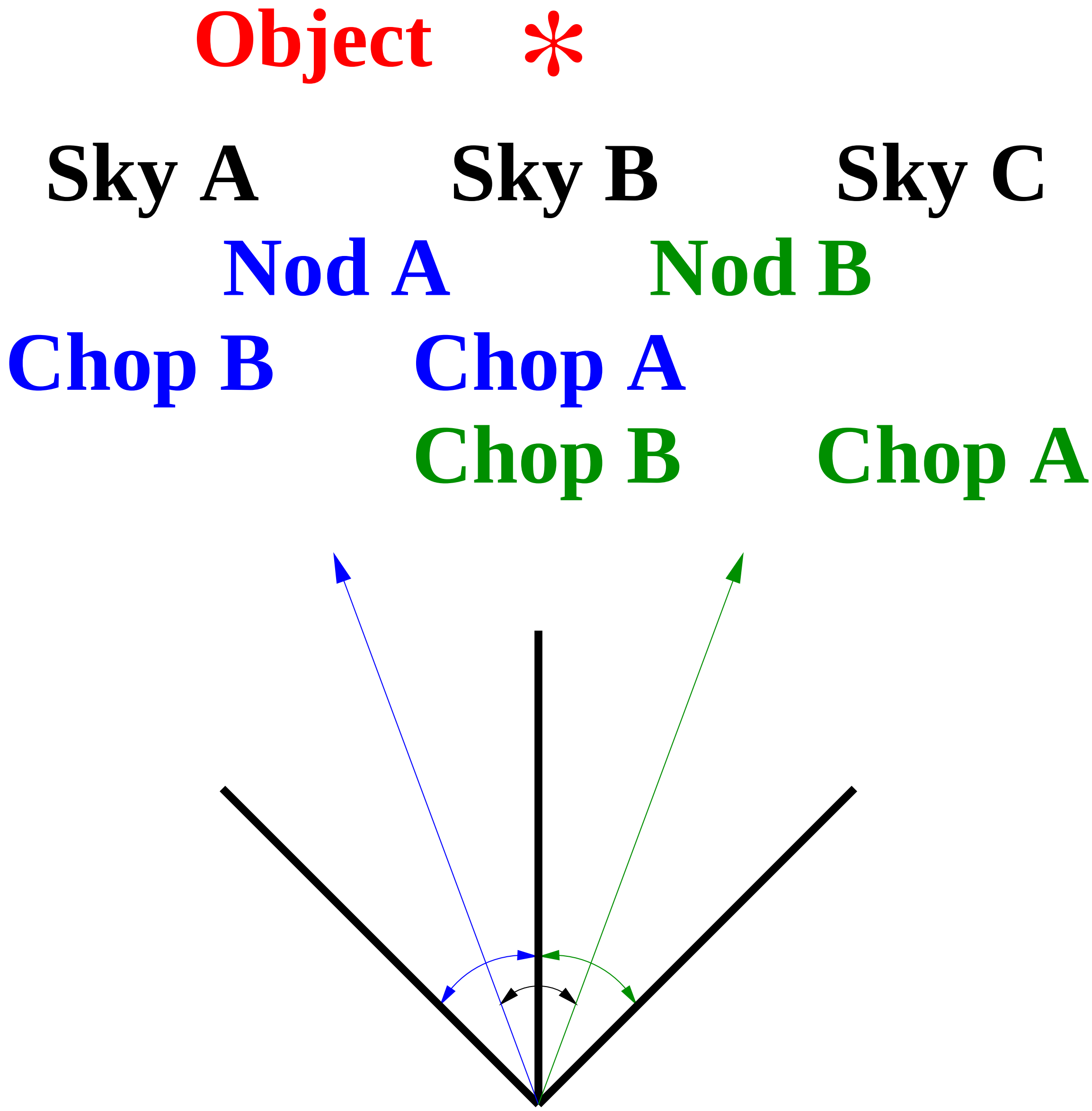

Figure 2 – The combined chop/nod situation is shown above where the blue and green arrows show the two telescope pointings in the nod positions while the black lines show the positions of the telescope in the different chop pointings. The three fields of view that result are labeled on the figure.

Assume that the telescope starts out in nod position A and an exposure is taken. The radiation components observed are

$$\begin{aligned}
\text{nod A}_1,\ \text{chop A} &\quad\rightarrow\quad \text{sky B}_1 + \text{telescope R}_1 + \text{source} \\
\text{nod A}_1,\ \text{chop B} &\quad\rightarrow\quad \text{sky A}_1 + \text{telescope L}_1
\end{aligned}$$

where the subscript 1 indicates this observation is taken at time 1, since later I will discuss what happens when the sky or the telescope contributions are changing with time: as one expects because the atmospheric temperature and the telescope temperature change with time.

The images at these two positions chop positions are taken for some time interval, and then the telescope is offset to nod position B and the observation is repeated–by assumption, for the same length of time. In this second nod position one observes

$$\begin{aligned}
\text{nod B}_2,\ \text{chop A} &\quad\rightarrow\quad \text{sky C}_2 + \text{telescope R}_2 \\
\text{nod B}_2,\ \text{chop B} &\quad\rightarrow\quad \text{sky B}_2 + \text{telescope L}_2 + \text{source}
\end{aligned}$$

so the source is now in the opposite chop position than it was previously.

Assume then that another observation is done in nod position B, and then the telescope is moved back to nod position A for a fourth observation of the same sort. From these one gets the same results except the subscripts are different to denote the different times:

$$\begin{aligned}
\text{nod B}_3,\ \text{chop A} &\quad\rightarrow\quad \text{sky C}_3 + \text{telescope R}_3 \\
\text{nod B}_3,\ \text{chop B} &\quad\rightarrow\quad \text{sky B}_3 + \text{telescope L}_3 + \text{source} \\
\text{nod A}_4,\ \text{chop A} &\quad\rightarrow\quad \text{sky B}_4 + \text{telescope R}_4 + \text{source} \\
\text{nod A}_4,\ \text{chop B} &\quad\rightarrow\quad \text{sky A}_4 + \text{telescope L}_4
\end{aligned}$$

What is done is to form the difference images between chop A and chop B for these four observations. Assuming the chop B observation is subtracted from the chop A observation what is obtained is

$$\begin{aligned}
\text{chop A}_1 - \text{chop B}_1 &= (\text{sky B}_1 - \text{sky A}_1) + (\text{telescope R}_1 - \text{telescope L}_1) + \text{source} \\
\text{chop A}_2 - \text{chop B}_2 &= (\text{sky C}_2 - \text{sky B}_2) + (\text{telescope R}_2 - \text{telescope L}_2) - \text{source} \\
\text{chop A}_3 - \text{chop B}_3 &= (\text{sky C}_3 - \text{sky B}_3) + (\text{telescope R}_3 - \text{telescope L}_3) - \text{source} \\
\text{chop A}_4 - \text{chop B}_4 &= (\text{sky B}_4 - \text{sky A}_4) + (\text{telescope R}_4 - \text{telescope L}_4) + \text{source}
\end{aligned}$$

If these observations were just added together the source contribution would (ideally) be perfectly eliminated because the source is positive in the differences for nod position A and negative for the differences in nod position B. One of course wants to detect the source so what one does instead is negate the differences for nod position B, so one has

$$\begin{aligned}
\text{chop A}_1 - \text{chop B}_1 &= (\text{sky B}_1 - \text{sky A}_1) + (\text{telescope R}_1 - \text{telescope L}_1) + \text{source} \\
\text{chop B}_2 - \text{chop A}_2 &= (\text{sky B}_2 - \text{sky C}_2) + (\text{telescope L}_2 - \text{telescope R}_2) + \text{source} \\
\text{chop B}_3 - \text{chop A}_3 &= (\text{sky B}_3 - \text{sky C}_3) + (\text{telescope L}_3 - \text{telescope R}_3) + \text{source} \\
\text{chop A}_4 - \text{chop B}_4 &= (\text{sky B}_4 - \text{sky A}_4) + (\text{telescope R}_4 - \text{telescope L}_4) + \text{source}
\end{aligned}$$

to work with. Now if one average these differences, one can see that the sky and telescope contributions will tend to cancel out, while the source contribution is included in all four terms and so will not cancel out. The expression for the average is

$$\begin{aligned}\text{averaged difference} &= (\text{sky B}_1 + \text{sky B}_2 + \text{sky B}_3 + \text{sky B}_4)/4 \\ &- (\text{sky A}_1 + \text{sky C}_2 + \text{sky C}_3 + \text{sky A}_4)/4 \\ &+ (\text{telescope R}_1 + \text{telescope R}_4 - \text{telescope R}_2 - \text{telescope R}_3)/4 \\ &+ (\text{telescope L}_2 + \text{telescope L}_2 - \text{telescope L}_1 - \text{telescope L}_4)/4 \\ &+ \text{source}\end{aligned}$$

which shows that the average we obtain contains the source contribution plus some contribution from the mean of *differences* of the sky and telescope emission seen in the set of observations.

Now consider a simple case, where the sky contributions and the telescope contributions are constant with time so for example sky $\text{A}_1 = \text{sky A}_4$, and so on, hence one can just drop all the subscripts that indicate which time things were observed. In such a situation

$$\begin{aligned}\text{averaged difference} &= (\text{sky B} + \text{sky B} + \text{sky B} + \text{sky B})/4 \\ &- (\text{sky A} + \text{sky C} + \text{sky C} + \text{sky A})/4 \\ &+ (\text{telescope R} + \text{telescope R} - \text{telescope R} - \text{telescope R})/4 \\ &+ (\text{telescope L} + \text{telescope L} - \text{telescope L} - \text{telescope L})/4 \\ &+ \text{source} \\ &= \text{sky B} - \frac{\text{sky A} + \text{sky C}}{2} + \text{source}\end{aligned}$$

so the telescope contribution is completely removed. The sky contribution will also cancel out provided that

$$\text{sky B} = \frac{\text{sky A} + \text{sky C}}{2} \quad ,$$

which is likely to be close to the real situation as long as positions A, B, and C are very close together in the sky. Note that it is not **certain** that the sky emission at position B equals the average of the sky emission at positions A and C. . .if there are strong gradients in the sky emission it may not cancel out properly no matter what chopping and nodding is being done. It is, however, observed that most of the time under good weather conditions the cancellation is excellent. Any linear gradients in the sky emission are removed by this chop/nod process. In bad weather the cancellation is observed to be poorer, and this can lead to a residual signal in a set of ABBA nod cycles.

The summary of the time independent case is that any linear gradients across the sky region being observed will cancel out in the chopping and nodding. Since the region of sky being observed is usually small and the chop distance is also small taking a linear approximation to the gradient normally works well.

When these assumptions hold one nod A observation and one nod B observation averaged together will cancel out the telescope and sky backgrounds. Why then have the equations been set up for a group of four nod observations? The answer is that having four nods in a group this way makes a difference if the sky or telescope contributions are changing with time. As long as the temperature of the atmosphere and of the telescope are constant with time, the above situation holds and the backgrounds one gets looking at the telescope or at the different parts of the sky are fixed. So they would cancel out nicely in an AB nod pair. However it is known that the temperatures of objects, including the atmospheric temperatures, are not constant, so one has to assume that all these background contributions are changing with time. Usually these changes are slow, but they depend on the weather and also on how the telescope temperature is changing. One nod A observation difference averaged with the negative of one nod B observation difference gives

$$\begin{aligned}\text{averaged difference} &= (\text{sky } \mathrm{B}_1 + \text{sky } \mathrm{B}_2)/2 \\ &- (\text{sky } \mathrm{A}_1 + \text{sky } \mathrm{C}_2)/2 \\ &+ (\text{telescope } \mathrm{R}_1 - \text{telescope } \mathrm{R}_2)/2 \\ &+ (\text{telescope } \mathrm{L}_2 - \text{telescope } \mathrm{L}_1)/2 \\ &+ \text{source}\end{aligned}$$

where one notes that the sky background is sampled at position C at one time and at position A at a different time. If we assume that at the initial time

$$\text{sky } \mathrm{B}_1 = \frac{\text{sky } \mathrm{A}_1 + \text{sky } \mathrm{C}_1}{2}$$

and also that at the later time

$$\text{sky } \mathrm{B}_2 = \frac{\text{sky } \mathrm{A}_2 + \text{sky } \mathrm{C}_2}{2}$$

these equations give the following for the sky contribution:

$$\begin{aligned}\frac{\text{sky } \mathrm{B}_1 + \text{sky } \mathrm{B}_2}{2} - \frac{\text{sky } \mathrm{A}_1 + \text{sky } \mathrm{C}_2}{2} &= \frac{(\text{sky } \mathrm{A}_1 + \text{sky } \mathrm{C}_1 + \text{sky } \mathrm{A}_2 + \text{sky } \mathrm{C}_2)}{4} \\ &\quad - \frac{(\text{sky } \mathrm{A}_1 + \text{sky } \mathrm{C}_2)}{2} \\ &= \frac{(\text{sky } \mathrm{A}_2 - \text{sky } \mathrm{A}_1)}{4} + \frac{(\text{sky } \mathrm{C}_1 - \text{sky } \mathrm{C}_2)}{4} \quad .\end{aligned}$$

These terms involving the different sky A and sky C values do not have to cancel out. They will cancel out if

$$\text{sky } \mathrm{A}_2 - \text{sky } \mathrm{A}_1 = \text{sky } \mathrm{C}_2 - \text{sky } \mathrm{C}_1 \quad ,$$

which would be the case if the emission from all three sky positions are changing the same way with time between the two observations. If that is the case one can use AB nodding and the sky contributions will again cancel out.

What is more commonly seen is that the sky emission from the two adjacent fields changes with time at slightly different rates. This is because the **fractional** change in brightness is more likely to be the same for the two sky positions than is the absolute rate of change of brightness at the two sky positions, so if the emission at position A changes by 1% over some given time the emission at position C will change by 1% as well. In this circumstance, if sky $\mathrm{A_1}$ is not the same as sky $\mathrm{C_1}$ then the changes in brightness will be different and the sky contributions will not cancel out.

This is where nodding in ABBA fashion gives one an advantage. Consider the average of four nods in ABBA pattern given above,

$$\begin{aligned}\text{averaged difference} &= (\mathrm{sky\ B_1} + \mathrm{sky\ B_2} + \mathrm{sky\ B_3} + \mathrm{sky\ B_4})/4 \\ &- (\mathrm{sky\ A_1} + \mathrm{sky\ C_2} + \mathrm{sky\ C_3} + \mathrm{sky\ A_4})/4 \\ &+ (\mathrm{telescope\ R_1} + \mathrm{telescope\ R_4} - \mathrm{telescope\ R_2} - \mathrm{telescope\ R_3})/4 \\ &+ (\mathrm{telescope\ L_2} + \mathrm{telescope\ L_3} - \mathrm{telescope\ L_1} - \mathrm{telescope\ L_4})/4 \\ &+ \mathrm{source} \quad .\end{aligned}$$

When the sky emission is changing with time can one still get cancellation of the background? One needs to have linear gradients across the sky at any time for cancellation in the simplest case. One can hope that these linear gradients will be preserved even if the value of the gradient changes with time. This requires that the changes in emission at any point are linear with time. Again this is not going to strictly be the case but when the changes are gradual it should be a good approximation. If one makes this assumption then for, say, sky position A we have that

$$\mathrm{sky\ A_2} = \mathrm{sky\ A_1} + \mathrm{a}\Delta \mathrm{t}_{2\rightarrow 1}$$

where a is some rate of change and $\Delta t_{2\rightarrow 1} = t_2 - t_1$. Assuming that the nods are equally spaced in time one would have

$$\mathrm{sky\ A_2} = \mathrm{sky\ A_1} + \mathrm{a}\Delta\mathrm{t} \quad \mathrm{sky\ A_3} = \mathrm{sky\ A_1} + 2\mathrm{a}\Delta\mathrm{t} \quad \mathrm{sky\ A_3} = \mathrm{sky\ A_1} + 3\mathrm{a}\Delta\mathrm{t}$$

and similar relations apply for the other quantities in the various equations, which is what will be assumed hereafter.

So consider the case where there are these linear time dependencies. If it is assumed that the sky background is linear over the three sky positions observed at each time so that

$$\begin{aligned}\mathrm{sky\ B_1} &= \frac{\mathrm{sky\ A_1} + \mathrm{sky\ C_1}}{2} \\ \mathrm{sky\ B_2} &= \frac{\mathrm{sky\ A_2} + \mathrm{sky\ C_2}}{2} \\ \mathrm{sky\ B_3} &= \frac{\mathrm{sky\ A_3} + \mathrm{sky\ C_3}}{2} \\ \mathrm{sky\ B_4} &= \frac{\mathrm{sky\ A_4} + \mathrm{sky\ C_4}}{2},\end{aligned}$$

that the sky emission values are changing linearly with time so that

$$\begin{aligned}
\text{sky } A_2 &= \text{sky } A_1 + a\Delta t_{2\to 1}\\
\text{sky } B_2 &= \text{sky } B_1 + b\Delta t_{2\to 1}\\
\text{sky } C_2 &= \text{sky } C_1 + c\Delta t_{2\to 1}\\
b &= (a+c)/2 \quad \text{to satisfy the equations above}\\
\text{telescope } R_2 &= \text{telescope } R_1 + r\Delta t_{2\to 1}\\
\text{telescope } L_2 &= \text{telescope } L_1 + l\Delta t_{2\to 1} \quad ,
\end{aligned}$$

and finally that the four observations are evenly spaced in time then the expression for the average becomes

$$\begin{aligned}
\text{averaged difference} = {} & b(\Delta t + 2\Delta t + 3\Delta t)/4\\
& - a(3\Delta t)/4 - c(\Delta t + 2\Delta t)/4\\
& + r(3\Delta t - 2\Delta t - \Delta t)/4\\
& + l(\Delta t + 2\Delta t - 3\Delta t)/4\\
& + \text{source}
\end{aligned}$$

(where Δt is the time between any two nod observations, as in the previous paragraph) or

$$\text{averaged difference} = 6\Delta t(b - (a+c)/2) + \text{source}$$

so if as assumed $b = (a+c)/2$ the sky emission term cancels out.

Compare this to the case where one nods in an ABAB pattern instead. In that case the averaged difference expression above becomes

$$\begin{aligned}
\text{averaged difference} = {} & b(\Delta t + 2\Delta t + 3\Delta t)/4\\
& - a(2\Delta t)/4 - c(\Delta t + 3\Delta t)/4\\
& + r(2\Delta t - 3\Delta t - \Delta t)/4\\
& + l(\Delta t + 3\Delta t - 2\Delta t)/4\\
& + \text{source}
\end{aligned}$$

(one just swaps all 3's and 2's in the equation above since now the second nod A and the second nod B are exchanged), which reduces to

$$\begin{aligned}
\text{averaged difference} = {} & (6b\Delta t - 2a\Delta t - 4c\Delta t)/4\\
& + (l - r)\Delta t/2\\
& + \text{source} \quad .
\end{aligned}$$

Using the equation $2b = a + c$ in the expression one gets the final result

$$\begin{aligned}
\text{averaged difference} = {} & (a - c)\Delta t/4\\
& + (l - r)\Delta t/2\\
& + \text{source}
\end{aligned}$$

in which the atmospheric and telescope contributions do not cancel out as well as they did in the ABBA nod pattern case. The reason for this is that one is always looking at one of the off source positions before the other, and similarly the right telescope position is always being looked at before the left telescope position. Using an ABBA nod pattern causes this to alternate: in one pair sky position A is looked at before sky position C, and in the next pair sky position C is looked at before sky position A, hence linear time gradients in these contributions cancel out. For this reason using an ABBA nod pattern should produce better results than an ABAB nod pattern.

In actual practise there are complications in all of these things. The telescope background tends to be slowly varying and thus either ABBA or ABAB nod patterns will remove that component quite well in most cases. Gradients in the sky emission tend to be associated with clouds so in clear conditions ABAB nodding will probably work well enough. Nevertheless an ABBA nod pattern is to be preferred to an ABAB nod pattern.